ADVANCES IN GEOMETRIC AND QUANTUM TOPOLOGY: OPEN PROBLEM SESSION

1. Volume and quantum invariants

Let K be a hyperbolic knot. The first two conjectures relate the volume of the complement of K to the rank of invariants like Khovanov homology and knot Floer homology.

Conjecture 1 (Champanerkar-Kofman-Purcell)**.**

$$\mathrm{Vol}(K) < 2\pi \log\left(\mathrm{rk}\ \mathrm{Kh}(K)\right).$$

Recall that $\mathrm{rk}\ \mathrm{Kh}(K) = \det K$ *when* K *is alternating.*

Conjecture 2. *There exists a constant* C *such that*

$$\mathrm{Vol}(K) < C\ \log\left(\mathrm{rk}\, H\widehat{F}K(K)\right)$$

Note that $\mathrm{rk}\, H\widehat{F}K(K) \leq \mathrm{rk}\ \mathrm{Kh}(K)$*.*

Conjectures 1 and 2 are known to be true for some families of knots.

There is a generalization in the direction of planar graphs. Let G be a planar graph and let K be an alternating link whose Tait graph is G. Then $\det(K)$ is the number of spanning trees of G. Write $\tau(G)$ for the number of spanning trees of G and Z_G for the spanning tree entropy; *i.e.*,

$$Z_G = \lim_{n\to\infty} \frac{\log \tau(G)}{|V(G)|}.$$

Conjecture 3. $\mathrm{Vol}(T^2 \times I \setminus L) \leq 2\pi Z_G$.

For the integral lattice, $2\pi Z_G = 10 v_{tet}$.

2. Rod complements in T^3

Some goals for studying rod complements in T^3.

(1) Find an algorithm to decompose into tetrahedra, or an alternate way to chop them into simpler pieces.
(2) Find good bounds on the volume that are based on rod parameters.

Question: is there an analogue of mutation, or does volume characterize rod complements?

(3) When are they arithmetic?
(4) Is there a Volume Conjecture for (stacks) of rods? – this would be a Reshetikhin-Turaev version, perhaps.

Some of the rod complements that have been studied are right-angled – valuable for volume bounds.

Question: Is there a knot that is right-angled?

3. Computational Complexity

Are the following problems in P, NP, NP-hard?

(1) the knot isotopy problem?
(2) 3-manifold homeomorphism problem?
(3) the unknotting number problem?

Some notes: Recognizing the unknot is in NP $\cap$ coNP.

- Passing between two triangulations of a 3-manifold is NP-hard.
- Jones polynomial is in #P-hard.
- Turaev-Reshetikhin-Viro is BQP.